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# Cost-Efficient and Scalable PMU Placement Strategies for Enhanced Power Grid Monitoring and Management

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**Abstract:** Phasor Measurement Units, also known as PMUs, were crucial in improving real-time monitoring and control of power systems, which in turn contributed to the grid's resilience and reliability. Nevertheless, determining the appropriate placement of PMUs (OPP) remained a challenge due to the need to balance the system's observability with deployment costs. During this research, a sustainable methodology for PMU placement was developed by including several different variables, including observability, cost-efficiency, and practicality. Using optimisation algorithms, strategic PMU locations were identified, and the use of previous fault data helped ensure the reliability of problem detection. The technique enabled a PMU deployment that is both flexible and scalable by considering a variety of grid designs, sizes, and financial constraints. Using case studies of actual power grids, their effectiveness was demonstrated compared to alternative solutions. The strategy optimised resource utilisation and operational expenses by reducing the number of PMUs to a minimum without compromising observability. This also enabled long-term adaptability by accounting for future grid growth.

**Keywords:** Cost-Efficient; Ant Lion Optimisation (ALO); Distribution Systems; Phasor Measuring Unit (PMU); Modelling and State Estimation; Transmission Systems; Fault Detection; Operational Costs; Power Grid Monitoring and Management.

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## 1. Introduction

Phasor Measurement Units (PMUs) are increasingly recognised as essential tools for modern power systems, providing real-time data that enhances the visibility and control of the electrical grid [16]. Their ability to measure voltage and current phasors with high precision and synchronicity enables operators to monitor the grid state in real time, which is critical for improving system reliability, stability, and fault detection [17]. As power systems evolve with the integration of renewable energy sources and smart grid technologies, the role of PMUs becomes even more vital. However, the challenge of optimally placing PMUs across a network remains a significant barrier, primarily due to the competing objectives of maximising system observability while minimising installation and operational costs [19].

The problem of optimal PMU placement (OPP) is inherently complex due to the need to balance multiple factors. Complete observability, where every bus in a power system can be monitored directly or indirectly, is often desirable but can lead to prohibitive costs if not managed carefully. PMUs, though valuable, are expensive to deploy and maintain, making cost considerations a critical component of the placement strategy [20]. Therefore, the objective of this research is to develop a sustainable methodology that achieves optimal placement by addressing three core factors: observability, cost, and real-world applicability, as demonstrated through case studies [18]. This methodology integrates mathematical optimisation techniques and machine learning algorithms to identify strategic PMU locations that maximise observability at minimum cost [21]. The traditional approaches to OPP, which rely heavily on deterministic methods or simple heuristic techniques, often overlook the adaptability required in modern power systems, especially those undergoing rapid technological changes and expansions. This research emphasises adaptability, aiming to create a flexible framework applicable to power systems of varying sizes and complexities [22].

One of the primary motivations for this research is the growing need for sustainable solutions in the energy sector. The current trends toward decentralisation and the integration of distributed energy resources (DERs), such as solar and wind, are increasing the complexity of managing power grids. The real-time monitoring capabilities of PMUs are indispensable in this context, as they provide critical data for managing intermittent energy sources and preventing grid instability [23]. However, these benefits must be achieved without imposing excessive financial burdens on utilities or grid operators, which underscores the need for cost-effective PMU placement solutions [26].

The methodology proposed in this research leverages historical fault data to improve the robustness of fault detection in the system [25]. By using machine learning, the system can learn from past faults and predict potential vulnerabilities in the grid, enabling preemptive actions. This predictive capability enhances the overall effectiveness of PMUs, providing an additional layer of reliability beyond simple real-time monitoring. Furthermore, case studies are an essential component of this research. By applying the proposed methodology to real-world power grids, the study seeks to validate its practicality and effectiveness in diverse scenarios. These case studies help bridge the gap between theoretical research and practical implementation, ensuring that the proposed PMU placement strategy is not only mathematically sound but also feasible for grid operators to adopt [24].

## 2. Review of Literature

Numerous solution approaches for the Phasor Measuring Unit (PMU) placement problem have been extensively documented in the literature, categorised into numerical/mathematical optimisation techniques, meta-heuristic algorithms, and hybrid methods [12]. A well-known approach for OPP is the dual search algorithm, which integrates a modified bisecting search with simulated annealing to reduce the number of PMUs needed [1]. However, because PMU locations are randomly selected, the bisecting search approach is computationally inefficient for solving the OPP problem.

Pal et al. [2], a line relay placement approach using "Integer Linear Programming (ILP)" to minimise total costs while ensuring adequate redundancy to handle device failures. Another ILP approach focuses on evaluating only probable configurations, each assigned a probability weight [3]. Despite significant advancements, these methods face challenges, including high computational demands and susceptibility to cumulative errors, which can affect their accuracy. Heuristic approaches, like the "Non-Dominated Sorting Genetic Algorithm (NSGA)", have been used to generate a Pareto-optimal solution for OPP [4]. However, this method does not always minimise the number of PMUs required for full system observability.

The greedy search algorithm, as shown by Zhao et al. [5], provides a quick dynamic response, making it well-suited for high-sampling-rate applications. Still, it prioritises ranking-based PMU placement over complete system observability. Meanwhile, the Lyapunov exponent-based OPP technique targets critical buses to achieve full network coverage [6], but it overlooks issues such as bad data detection. Muscas et al. [7] studied the impact of uncertainty components on voltage profiles in distribution systems using Weighted Least Squares (WLS) estimators. However, this method is solely reliant on weight estimates.

To address faulty measurement data, "Principal Component Analysis (PCA)" and "Cumulative Sum (CUSUM)-based Generalised Likelihood Ratio (GLR)" algorithms have been employed for bad data detection and cyber-attack detection in both centralised and distributed systems [8]. However, the arrangement of power systems presents random errors in state estimation. The security of transmitted data remains a significant concern in state estimation. A recent review of secure state estimation against PMU placement attacks highlighted a location identification scheme that efficiently detects likely line outages by considering phasor angles, bad data measurements, and a multi-hypothesis test with a limited number of PMUs [13].

However, these studies indicate that complete system observability is not always achieved. In addition, voltage stability is analysed using the line stability index, specifically the FVSI, to assess the stress level of transmission lines [14]. With growing electricity demand and increased power transfers between utilities, it is critical to manage communication resources efficiently. Weak buses can also be identified using modern advanced power electronic devices. Although such devices can enhance system utility [15], they do not significantly improve power flow control when implemented on weak buses [9]. Considering the limitations of the algorithms as mentioned above, the Binary Coded Ant Lion Optimisation (BCALO) toolbox has been integrated into the PMU placement problem to optimise PMU placement in both transmission and distribution systems [10].

## 3. Problem Identified

Before engaging in this research, the student had limited coursework experience in state estimation, where foundational concepts and power system network analysis were introduced in later courses. Additionally, the student had developed a co-simulation framework to assess the cost of communication equipment during lab experiments [11]. Building on this foundational understanding of state estimation and incorporating applications that aligned with their prior co-simulation work, the student was assigned the following tasks: (i) test a PMU to understand its operation, (ii) create documentation to guide other students in using this equipment, and (iii) assess whether PMUs are sensitive to failures within the power system network.

### 3.1. Objective and Contribution

This paper introduces a sustainable methodology for optimal PMU placement in power systems, integrating observability, cost efficiency, and sustainability considerations into a unified framework. By leveraging optimisation techniques, the approach minimises the number of PMUs required, ensures system resilience through redundancy, and maintains critical observability during outages. Unique contributions include incorporating lifecycle costs and environmental impact, alongside traditional metrics, to align with global sustainability goals.

Validated through IEEE test systems and real-world case studies, the methodology demonstrates significant cost savings and improved reliability compared to conventional techniques. Additionally, the paper provides open-source tools to facilitate further research and practical application. The purpose of these tasks was to achieve the following learning objectives:

- To develop a comprehensive PMU placement model for both transmission and distribution networks, taking into account cost, coverage, and voltage stability.
- To understand the effects of PMU failures and multiple line outages in transmission and distribution networks, and how these can be used to communicate data to the power system control centre over wide areas.
- To mitigate data transmission failures within the network and to broaden the application of meta-heuristic algorithms in solving the PMU placement challenge.

## 4. Mathematical Modelling for Transmission and Distribution Systems

### 4.1. Normal System Operation

For an N-bus system with m measurements of voltage and current phasors, the linear matrix equation that relates these measurements to the system's state vector can be written as:

$$\Delta Z = h_{nk}\Delta x + e_{nk} \quad (1)$$

If the rank of $h_{nk}$ If the rank of the state vector equals the dimension, the solution is unique, indicating that the network is observable. The gain matrix, J(x), is formulated as:

$$J(x) = (\Delta Z - h_{nk}\Delta x)^T R^{-1}(\Delta Z - h_{nk}\Delta x) \quad (2)$$

The OPP problem can be expressed as a nonlinear WLS minimisation model, which is represented as:

$$\min\ (x) = \sum_{i=1}^{N} W_i * X_i^2 \quad (3)$$

### 4.2. Impact of Zero-Injection Buses (ZIBs)

This method accounts for the presence of ZIBs when determining PMU placement, minimising the number of PMUs required for full observability of the power network. The objective function is:

$$f(x) = \sum_{i,j}^{N} C_{ij} x_j > k_i \quad (4)$$

**4.3. Single PMU Failure**

To enhance system reliability in the event of a single PMU or communication link failure, it is crucial to deploy additional PMUs. The goal of the objective function is

$$f(x) = \sum_{i,j}^{N} C_{ij} x_j > k_i \quad (5)$$

**4.4. Multiple Line Outages**

The primary aim of OPP is to reduce the number of deployed PMUs while ensuring full system observability, even in scenarios with multiple line outages.

$$f(x) = \sum_{i,j}^{N} C_{ij} x_j > k_i \quad (6)$$

Where

$$X_i = \begin{cases} 1 \text{ when PMU is installed at bus i} \\ 0 \qquad \text{otherwise} \end{cases} \quad (7)$$

This objective function reflects the weighted total of the PMUs included in the analysis. The weights are determined by the total number of lines connected to bus iii, plus one, representing the number of PMU measurement channels. A binary connectivity matrix is created from the system line data to illustrate these connections.

$$C_{ij} = \begin{cases} 1, \qquad \text{if } i = j \\ 1, \text{when buses i and j are connected} \\ 0, \qquad \text{otherwise} \end{cases} \quad (8)$$

**4.5. Fast Voltage Stability Index (FVSI)**

Minimising FVSI can be articulated as:

$$\min \sum FVSI_{ij} k_j \quad (9)$$

where,

$$k_j = \begin{cases} 1, \text{when a PMU is installed at bus i} \\ 0, \qquad \text{otherwise} \end{cases} \quad (10)$$

**4.6. Constraints**

According to equation (3), the optimal allocation of PMUs under normal operating conditions is governed by the following constraint:

$$\text{Min } \sum_{i=1}^{N} X_i \geq 1 \quad (11)$$

Similarly, for optimal allocation with maximum redundancy, the constraint is:

$$\sum_{i=1}^{N} X_i \geq 1 \quad (12)$$

In scenarios involving multiple line outages, affected lines cannot securely transmit data. To maintain continuous observability, every bus affected by an outage must be monitored by at least 2 independent PMUs on different lines. To ensure complete system observability under all operational conditions, including during line outages, the requirement for redundancy is established as follows:

$$k_i = \begin{cases} 1 \text{ for normal operating condition} \\ 2 \qquad\quad \text{for multiple line outages} \end{cases} \tag{13}$$

According to equation (6), a power system is considered fully observable under normal conditions when ki=1, indicating that each bus is monitored at least once. To achieve complete observability during multiple line outages, each bus must be observed at least twice, which corresponds to ki=2. The OPP seeks to identify strategically important, heavily loaded, and future-expansion buses for the installation of PMUs. Equations (3)–(13) are derived using the methodologies and principles outlined in references [16]–[17]; [25]. These equations serve as the foundation for the PMU placement framework in the proposed work.

### 4.7. Voltage Stability Equations

The FVSI was derived from the quadratic voltage equation at the receiving bus in an N-bus system. The current flow in a two-bus system is given by:

$$I = \frac{E_i\angle 0 - E_j \angle \theta}{Z} \tag{14}$$

The apparent power at $j^{th}$ bus is

$$S_j = E_j I^* \tag{15}$$

Rearranging the (15) gives

$$I^* = \left(\frac{S_j}{E_j}\right) = \frac{P_j - jQ_j}{E_j\angle -\theta} \tag{16}$$

Equating the equations (14) and (16)

$$\frac{E_i\angle 0 - E_j\angle \theta}{Z} = \frac{P_j - jQ_j}{E_j\angle -\theta} \tag{17}$$

This results in:

$$E_i E_j \,\mathrm{ccos}\, \theta - E_j^2 = Z\left(P_j - jQ_j\right) \tag{18}$$

By separating real and imaginary parts and rearranging, we obtain:

$$E_i E_j \cos \theta - E_j^2 = RP_j + XQ_j \tag{19}$$

Finally, the FVSI is defined as:

$$FVSI_{ij} = \frac{4Z^2 Q_j}{E_i^2 X} \tag{20}$$

The FVSI indicates the condition of a line. If the FVSI is low, the line is in good condition, while a higher FVSI (closer to 1) indicates poor stability. PMU placement is crucial to maintaining the FVSI below 1, ensuring maximum voltage stability. The above problem formulation applies to transmission system state estimation. It can be adapted to distribution system state estimation by partitioning the transmission system into clusters for secure data transfer under varying conditions.

## 5. Model implementation in ALO

### 5.1. ALO in brief

The Ant Lion Optimiser (ALO) was a meta-heuristic algorithm inspired by the natural interactions between ants and antlions, particularly the way antlions capture their prey. In their life cycle, antlions undergo two primary stages: larval and adult. During the larval stage, the antlion created a cone-shaped pit in the sand by moving in circular motions and ejecting sand. After constructing the trap, the larva concealed itself at the bottom of the cone, waiting for prey to fall in. The cone's sharp edges allowed insects to slide easily to the bottom, where the antlion awaited. Upon detecting prey in the trap, the antlion cleverly tossed sand towards the edges of the pit to drive the prey further down. After consuming the prey, the antlion discarded the remains and modified the trap for future hunts. These behaviours were mathematically modelled and extensively documented in the literature. The key phases of the hunting process in the ALO algorithm include random ant movement, trap construction, pray ensnarement, prey capture, and trap reconstruction.

### 5.2. Why BCALO?

The goal of optimisation problems was not merely to identify the extreme solution but to accurately trace their paths across the solution space. In the context of complex, dynamic, and widespread real-world challenges, an excessive reliance on existing search algorithms proved counterproductive, diminishing the potential for exploration. While this tendency may have been acceptable in standard problems, dynamic optimisation scenarios necessitated a shift in focus: attempts to manipulate an extreme often moved away from the intended outcome. To address the balance between exploration and exploitation, a novel algorithm was proposed for optimisation in dynamic binary environments. The Binary Coded Ant Lion Optimiser emulated certain behaviours of social insects, such as laying pheromone trails on a graph to guide the search.

These trails highlighted pseudo-solutions that gained relevance to the optimisation problem at hand. In this binary landscape, the installation of Phasor Measurement Units at buses was represented using binary values (0 and 1). Given that the Ant Lion Optimiser typically operated using real-coded schemes, the sigmoid function was employed to convert real values into binary by calculating the probability for each ant's position.

If this probability was lower than a randomly generated number between 0 and 1, the unit was installed at the bus (represented by 1); otherwise, it was not (represented by 0). As the model integrated two objectives, the fuzzy decision-making mechanism was combined with the Binary Coded Ant Lion Optimiser to manage them. A linear membership function was utilised, and the procedure for solving the Multi-Objective Placement Problem was outlined accordingly.

### 5.3. Step-by-Step BCALO Procedure:

- **Step 1:** Defined the input data and initialised the parameters for the Ant Lion Optimiser, including the number of search agents and the maximum number of iterations. In this framework, the ants and antlions represented solution vectors, with their positions in the search space corresponding to the output of each control variable in the problem of Phasor Measurement Unit placement.
- **Step 2:** Randomly generated an initial population of ants and antlions based on the total number of search agents and computed their fitness values. The fittest antlion from this initial population was selected as the elite solution.
- **Step 3:** Conducted the multi-objective optimisation strategy.
    - **Step 3.1:** Calculated the objective function values using established equations.
    - **Step 3.2:** Normalised these objective values employing a fuzzy decision-based approach.
    - **Step 3.3:** Computed the normalised membership values and identified the best compromise solution.
- **Step 4:** Designate the best compromise solution as the elite solution.
- **Step 5:** Incremented the iteration counter by one.
- **Step 6:** Applied a roulette wheel selection method to choose an antlion, where the stronger the antlion, the higher its likelihood of capturing more ants. This involved creating traps for the ants.
- **Step 7:** Adjusted the lower and upper bounds by contracting the search space.
- **Step 8:** Performed a random walk and normalised it. The search space was reduced by shifting the range of control variables toward the selected antlion.
- **Step 9:** Updated the ants' positions, saving the updated solutions as they moved around the chosen antlion.
- **Step 10:** Implemented the multi-objective strategy again.
- **Step 11:** Replaced an antlion with its corresponding ant if the ant exhibited better fitness.
- **Step 12:** Updated the elite solution if the antlion's fitness exceeded the current solution.
- **Step 13**: Stopped the search if the iteration count reached the predefined maximum number of iterations; otherwise, returned to Step 5.

### 5.4. Binary Coded Ant Lion Optimisation for Solving the Proposed Model

In Ant Lion Optimisation, the fundamental phases of hunting include random ant movement, trap construction, prey entrapment, and trap reconstruction. In the proposed EROSE model, binary values (0 and 1) represented the status of Phasor Measurement Unit installation at various buses. As Ant Lion Optimisation was primarily a real-coded algorithm, a sigmoid function was used to calculate the probability of each ant's position, effectively converting the real-coded representation into a discrete space. If the calculated probability was less than a randomly generated number (between 0 and 1), the PMU installation status at a bus was set to 1; otherwise, it was set to 0.

### 5.5. Enhanced Ant Lion Optimisation Algorithm: Why EALO?

A recently introduced optimisation algorithm, known as Ant Lion Optimisation, is inspired by the hunting behaviour of ant lions. It has shown effectiveness in addressing optimisation problems due to its simplicity, minimal parameter requirements, and high computational speed. Nevertheless, like other stochastic algorithms, Ant Lion Optimisation was prone to premature convergence and to trapping in local optima, particularly in complex or large-scale problems. To mitigate these drawbacks, an enhanced version, Enhanced Ant Lion Optimisation, was developed to address the PMU placement problem in Distribution System State Estimation while accounting for multiple line outages. Several enhancements were incorporated into the Enhanced Ant Lion Optimisation algorithm.

First, Particle Swarm Optimisation was introduced to refine and update the positions of the ant lion population in each iteration, following the search process of Ant Lion Optimisation. Next, a chaotic mutation operator —specifically, a logistic map —was applied to the elite, increasing the likelihood of escaping local optima. Finally, a combined serial-parallel approach for generating mutant particles was proposed, enhancing population diversity without requiring additional mutation iterations. The primary objective was to minimise the weighted objective function. Simulation results indicated that Enhanced Ant Lion Optimisation was an efficient and accurate method compared to other optimisation algorithms. It exhibited strong convergence characteristics and high stability, making it a suitable choice for solving optimisation problems related to PMU placement in Distribution System State Estimation under cascaded outage conditions.

### 5.6. Addressing Premature Convergence in ALO

While ALO offers fast computation, efficiency, and solid convergence, it faces challenges of premature convergence and local optima in complex optimisation problems. Several enhancements have been introduced to improve the algorithm's performance and precision. These include combining ALO with PSO, incorporating a chaotic mutation operator, and using a serial-parallel method to generate mutant particles. The PSO component enhances EALO's search capabilities by combining the exploration and communication traits of ALO and PSO, thereby improving search efficiency. In the chaotic mutation operator, a logistic map is integrated to modify the elite's position, increasing the likelihood of escaping local optima. The mathematical expression for this process is as follows:

$$\delta_{M+1} = \gamma \times \delta_M \times (1 - S_M) \qquad 0 < \gamma \leq 4 \tag{21}$$

Depending on the value of γ, the system exhibits different behaviours. For $0<\gamma<3$, the system is non-chaotic. At $\gamma > 3$, it begins to cycle, and at $\gamma = 4$, the system becomes chaotic. The value $\gamma = 4$ corresponds to the point at which the logistic map exhibits its most extreme chaotic behaviour. Values of $\gamma>4$ could lead to results outside the valid range of δ, which is typically constrained between 0 and 1, ensuring the system remains within a physically meaningful range; similarly, γ cannot be zero because it would make the system static, halting any meaningful evolution or mutation in the optimisation process.

In the serial-parallel combined method for obtaining mutant particles, this strategy ensures randomness and increases particle diversity without additional mutation iterations. The new objective is to minimise the weighted least squares error between the actual and estimated output vectors, which is defined as:

$$\Psi_{IOF}(\theta) = \sum_{k=1}^{L} \sum_{j=1}^{n} w_j \left(z_j(k) - \check{z}_j(k)\right)^2 \tag{22}$$

**The EALO algorithm followed these steps:**

- **Initialisation:** The initial positions of the ants and ant lions were established.
- **Fitness Calculation:** The fitness of each ant lion was computed.
- **Random Walks:** An ant lion was selected using the Roulette wheel method, and random walks were used to update the ants' positions.

- **Position Update:** The random walk process was repeated for all ants until their positions were updated.
- **Elite Comparison:** The positions of the ant lions were updated, and the fitness of the new ant lions was compared with that of the elite. If an ant lion's fitness surpassed that of the elite, the elite was replaced with the new ant lion's position.
- **PSO Optimisation:** Particle Swarm Optimisation (PSO) was employed to search for better ant lions and update the elite.
- **Chaotic Mutation:** Chaotic mutation was applied to the elite, generating mutant particles via a serial-parallel combination.
- **Repeat:** Steps 3 to 7 were repeated until the stopping criteria were satisfied.

## 6. Results and Discussion

Following the PMU model development and implementation, it was integrated with the ALO toolbox (in the MATLAB environment), and several case studies were carried out under various operating conditions. Indulgent interpretations of different test systems provide deep understandings into the varying complexities of power grids, enabling tailored PMU placement strategies.

Each test system —from IEEE standard networks to real-world grids —presents unique challenges in scale, topology, and operational constraints. By interpreting these systems, researchers can refine their methodologies, ensuring robustness across diverse scenarios. The comparison of test results highlights the adaptability and scalability of the proposed PMU placement techniques.

Ultimately, these interpretations guide the development of more resilient and efficient power grid monitoring solutions.

- **Case 1:** Neglecting the effect of ZIBs
- **Case 2:** Consequence of ZIB
- **Case 3:** System operation considering a single PMU loss
- **Case 4:** System operation considering a single line outage
- **Case 5:** The effect of multiple line outages (Distribution systems)
- **Case 6:** Single-Objective PMU Placement – Focused on minimising either the total installation cost (FTcost) or the Fast Voltage Stability Index (FFVSI).
- **Case 7:** Multi-Objective PMU Placement (MOPPMU) – Aiming to simultaneously minimise both FTcost and FFVSI.
- **Case 8:** Assessment of the Impact of Data Attacks – Evaluates how data attacks affect the placement and functionality of PMUs.

### 6.1. Case 1: Ignoring the Influence of Zero Injection Buses (ZIBs)

In a typical scenario, all bus connections are operational, and the Phasor Measurement Units (PMUs) installed at each bus operate without issue. A PMU located at a specific bus has sufficient input channels to measure the bus voltage phasor and the current phasors of all connected lines. Consequently, it can also calculate the voltage phasors of all buses linked to that particular bus.

A bus is considered observable if a PMU is installed on it or on a neighbouring bus. This case study presents simulation results obtained while disregarding the impact of Zero Injection Buses (ZIBs). Table 1 presents the optimal locations for measuring units across various test systems. The total cost, denoted FCost, is calculated using Equation (2.1), which incorporates the number of PMUs and the costs associated with the PMUs and communication channels.

**Table 1:** Results of BCALO under normal operating conditions: case 1

| Test System | BCALO | $F_{Cost}$ ($) | Average Availability | $F_{Tcost}$ ($) | $F_{SORI}$ |
|---|---|---|---|---|---|
| IEEE- 14 Bus | 2,6,7,9 | 122K | 0.99 | 174 | 19 |
| IEEE- 30 Bus | 1,2,6,9,10,12,15, 19,25, 27 | 290K | 0.99 | 225 | 50 |
| NE 39 Bus | 2,6,9,10,12, 14,17,19,20, 22,23,25,29 | 377K | 0.99 | 308.9 | 51 |
| IEEE- 57 Bus | 1,4,9,14,20, 22,25,27,29, 32,36,39,41, 45, 48,51,54 | 511K | 0.99 | 365.4 | 72 |
| IEEE- 118 Bus | 3,5,9,12,15, 17,20,23,25, 29,34,37,40, 45,49,53,56, 62,64,68,71, 75, 77, 80, 85,86,91,94, 102, 105, 110,115 | 1006K | 0.99 | 2765 | 163 |

| | | | | | |
|---|---|---|---|---|---|
| Indian Utility 62 (Distribution System) | 1,4,8,12 14,21,25,30, 34,41,42,46, 48,51,56,61 | - | - | - | 81 |
| Tamil Nadu Power Grid | 6,7,9,12,19, 27,30,33,35,47, 48,50,54,58,60, 63,67,73,75 | 603K | 0.99 | 601.9 | 89 |

### 6.2. Case 2: Impact of ZIBs

This scenario examines the influence of ZIBs on PMU placement, aiming to minimise the total number of PMUs required for complete observability of the power network. As indicated in Tables 1 and 2, the overall system cost in Case 2 is lower than in Case 1, due to the reduced number of PMUs required. For example, in the IEEE-14 bus system, the count of PMUs decreases from 4 in Case 1 to 3 in Case 2 (Table 2).

**Table 2:** Optimal locations of PMUs for various test systems, including the effect of ZIBs: case 2

| Test System | BCALO | $F_{Cost}$ (\$) | Average Availability | $F_{Tcost}$ (\$) | $F_{SORI}$ |
|---|---|---|---|---|---|
| IEEE-14 Bus | 2,6,9 | 99K | 0.99 | 126.729 | 16 |
| IEEE-30 Bus | 1,2,10,12,15, 20 | 180K | 0.99 | 189.488 | 43 |
| NE 39 Bus | 3,8,12,16,20, 23,25,29 | 247K | 0.99 | 320.062 | 38 |
| IEEE-57 Bus | 1,9,14,20,25, 27,29, 32,41,51,54 | 331K | 0.99 | 359.872 | 61 |
| IEEE-118 Bus | 3,12,15,17, 20,23,25,29, 34,40, 45,49,53,56, 62,75,77,80, 85,86,91,94, 102,105, 110,115 | 817K | 0.99 | 1127.62 | 153 |

### 6.3. Case 3: Loss of a Single PMU

The failure of a measurement unit not only renders the bus on which the PMU is installed unobservable but also affects the observability of adjacent buses. Thus, it is crucial to evaluate the impact of losing a single PMU when determining its placement. Table 3 presents the PMU locations under a single PMU loss, along with the corresponding total cost and measurement redundancy results. Table 3 shows that more than half of the buses require PMUs to establish a resilient measurement system.

**Table 3:** Optimal placement of PMUs in case of single PMU loss: case 3

| Test System | BCALO | $F_{Cost}$ (\$) | Average Availability | $F_{Tcost}$ (\$) | $F_{SORI}$ | CPU Time |
|---|---|---|---|---|---|---|
| IEEE-14 Bus | 1, 2, 3, 6, 7, 8, 9, 10, 13 | 202K | 0.99 | 517.07 | 32 | 0.071 |
| IEEE-30 Bus | 2,3,4,5,6,9,10,11,12,13,15,17,18,19,21,24, 25,26,27, 28,29 | 516K | 0.99 | 1475.4 | 84 | 0.126 |
| NE 39 Bus | 1,2,4,6,8,9,10,12,13,14,16,17,18,19,20,21,22,23,25 ,27,28,29,30,31,32,33,34,35,36,37,38 | 734K | 0.99 | 1518.5 | 102 | 0.235 |
| IEEE-57 Bus | 1,3,4,6,9,11,12,15,19,20,22,24,25,27,28,29,30,32,3 3,35,36,37,38,41,45,46,47,50,51,53, 54, 56,57 | 834K | 0.99 | 1886.5 | 130 | 0.394 |
| IEEE-118 Bus | 1,3,5,6,9,10,11,12,15, 17,19,21,22,25,27,28,30,31, 32,34,35,37,40,42,44,45,46,49,50,51,52,54,56,59,6 1,64,66,67,68,70,71,72,73,75,76,77,78,80,84,85,86 ,87,89,91,92,94,96,100,101,105,106,108,110,111,1 12,115,116,117 | 1224K | 0.99 | 3187.3 | 305 | 0.616 |

### 6.4. Case 4: Outage of a Single Line

The effectiveness of the proposed model is further highlighted when considering the impact of a single line outage. To ensure full observability during such an outage, every line must remain observable even if one line fails. The OPP is assessed by systematically removing each line from the network, with results shown in Table 4. Table 4 indicates that additional PMUs are required (beyond normal operating conditions) to achieve full network observability after a line outage. While the number of PMUs needed in this scenario is greater than in standard operation, it remains lower than that required in Case 3.

**Table 4:** Results of optimal locations of PMUs in case of single line outage: case 4

| Test System | BCALO | $F_{Cost}$ ($) | Average Availability | $F_{Tcost}$ ($) | $F_{SORI}$ | CPU Time |
|---|---|---|---|---|---|---|
| IEEE-14 Bus | 2,4,5,6,9,11,13 | 182K | 0.99 | 176.3677 | 34 | 0.102 |
| IEEE-30 Bus | 2,4,6,7,10,12,15,17,18,24,30 | 304K | 0.99 | 981.0301 | 54 | 0.290 |
| NE 39 Bus | 2,6,8,13,14,15,17,20,21,24,25,26,28, 34,36,37,38 | 445K | 0.99 | 2223.221 | 58 | 0.421 |
| IEEE-57 Bus | 1,6,8,13,14,15,19,27,29,30,32,33,38,41,49,51,53,55, 56 | 536K | 0.99 | 3789.285 | 87 | 0.662 |
| IEEE-118 Bus | 2,7,10,11,12,15,17,19,21, 23,24,25,27,29,32,34,36, | 718K | 0.99 | 4738.8 | 247 | 0.904 |

### 6.5. Case 5: Multiple Line Outages (For Distributed Systems)

Disturbances are introduced into the system through various line outages, which may hinder the achievement of complete observability. This lack of observability can lead to line overloads, potentially resulting in system failure. Furthermore, such disruptions compromise the secure transfer of data.

To address this issue, additional PMUs must be installed in the system. Table 5 provides an overview of multiple line outages. In this context, PMU placement for distributed system state estimation (DSSE) is initiated, successfully ensuring system observability even with multiple lines removed.

**Table 5:** Optimal PMU placement considering multiple line outages for the IEEE 14 bus system

| Test System | Line outages | Locations | SMRI |
|---|---|---|---|
| IEEE-14 Bus | 2 -3, 3-4, 4-7, 6-11, 7-8, 10-11 | 2, 3, 5, 6, 8, 9, 11, 13 | 25 |
| IEEE-30 Bus | 3-4, 4-6, 6-8, 9-11, 4-12, 12-15, 16-17, 10-20, 10-22, 22-24, 25-26, 27-29, 2-28 | 2, 3, 6, 8, 10, 11, 12, 14, 18, 20, 22, 24, 26, 27, 28, 29 | 45 |
| IEEE-57 Bus | 3-4, 6-7, 9-10, 9-13, 1-15, 3-15, 5-6, 11-13, 12-17, 18-19, 21-22, 24-25, 27-28, 7-29, 30-31, 34-32, 35-36, 37-39, 11-41, 38-44, 46-47, 49-50, 29-52, 11-43, 47-49, 33-34, | 1, 2, 4, 5, 8, 9, 12, 13, 15, 19, 21, 24, 27, 28, 29, 30, 32, 34, 36, 38, 39, 41, 43, 45, 46, 47, 50, 51, 53, 54, 56, 57 | 95 |
| IEEE- 118 Bus | 3-5, 8-9, 4-11, 3-12, 13-15, 15-17, 19-20, 21-22, 23-25, 27-28, 30-17, 17-31, 27-32, 35-37, 38-37, 39-40, 43-44, 45-46, 47-49, 45-49, 49-51, 49-54, 54-56, 50-57, 54-59, 55-59, 60-61, 63-59, 49-66, 66-67, 49-69, 70-71, 70-74, 76-77, 78-79, 79-80, 82-83, 85-86, 85-89, 89-92, 92-94, 82-96, 80-98, 95-96, 100-101, 100-103, 103-105, 105-107, | 1, 4, 5, 10, 11, 14, 15, 17, 19, 21, 23, 25, 26, 29, 35, 37, 41, 43, 45, 47, 49, 51, 54, 57, 59, 62, 63, 65, 67, 72, 73, 75, 76, 78, 79, 81, 82, 84, 87, 88, 91, 93, 95, 97, 98, 99, 101, 104, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117 | 165 |
| IU 62 | 108-109, 110-111, 17-113, 114-114 | 2, 4, 5, 6, 8, 9, 11, 12, 13, 14, 16, 18, 19, 21, 23, 26, 27, 28, 30, 32, 33, 35, 36, 37, 39, 42, 43, 45, 46, 49, 52, 53, 54, 56, 58, 59, 60, 62 | 109 |
| IU 146 | 1-4, 1-6, 2-3, 4-15, 13-14, 11-10 | | |

### 6.6. Case 6: Single Objective PMU Placement: Minimisation of FTcost or Minimisation of FFVSI

In this case, a voltage stability analysis is conducted, ranking the load buses by their FVSI values in ascending order. Table 6 presents the arrangement of the weaker buses.

### 6.7. Case 7: Multi-Objective PMU Placement (MOPPMU): Minimisation of FTcost and FFVSI

The improved outcomes, as outlined in Table 8, incorporate both FTcost and FVSI, presenting a constrained optimisation problem. Here, both FTcost and FVSI are optimised simultaneously. It is evident from Tables 7 and 8 that the number of PMUs and the weak buses attain a balanced configuration between the two objectives considered.

### 6.8. Case 8: Impact of Data Attacks

One key technique for simulating a series of attacks within the modelled network is to establish an attack scenario. This application allows the network to evolve through event simulations only when the attack scenarios are thoroughly defined.

**Table 6:** PMU Placement with FVSI-based voltage stability analysis for various test systems

| **Load Bus** | **$Q_{max}$ (p.u)** | **FVSI** | **Critical line** |
|---|---|---|---|
| IEEE 14 Bus System | | | |
| 9 | 0.43 | 0.99 | 4-9 |
| 13 | 0.54 | 1.00 | 12-13 |
| 14 | 0.54 | 0.99 | 13-14 |
| 12 | 0.83 | 1.00 | 6-12 |
| 11 | 0.91 | 0.99 | 10-11 |
| 6 | 0.99 | 0.99 | 5-6 |
| 4 | 1.16 | 0.99 | 3-4 |
| 5 | 1.19 | 1.00 | 1-5 |
| 3 | 1.28 | 0.98 | 2-3 |
| 10 | 1.53 | 0.99 | 9-10 |
| **IEEE 30 Bus System** | | | |
| 30 | 0.25 | 0.98 | 27-30 |
| 26 | 0.29 | 0.95 | 25-26 |
| 29 | 0.33 | 0.96 | 27-29 |
| 10 | 0.45 | 1.00 | 6-10 |
| 15 | 0.50 | 0.98 | 14-15 |
| 24 | 0.57 | 0.99 | 23-24 |
| 14 | 0.72 | 1.00 | 12-14 |
| 18 | 0.73 | 0.99 | 15-18 |
| 23 | 0.77 | 0.99 | 15-23 |
| 20 | 0.84 | 1.00 | 10-20 |
| 19 | 0.88 | 0.99 | 18-19 |
| 17 | 0.92 | 1.00 | 16-17 |
| 16 | 0.93 | 0.99 | 12-16 |
| 12 | 0.96 | 1.00 | 4-12 |
| 28 | 1.04 | 0.99 | 8-28 |
| 22 | 1.05 | 0.99 | 10-22 |
| 4 | 1.33 | 0.99 | 2-4 |
| 3 | 1.42 | 0.99 | 1-3 |
| 21 | 1.60 | 0.99 | 10-21 |
| 7 | 1.71 | 1.00 | 5-7 |
| 2 | 4.39 | 1.00 | 1-2 |

| **Load Bus** | **$Q_{max}$ (p.u)** | **FVSI** | **Critical line** |
|---|---|---|---|
| IEEE 57 Bus System | | | |
| 57 | 0.17 | 0.95 | 39-57 |
| 56 | 0.19 | 0.96 | 40-56 |
| 32 | 0.19 | 0.95 | 31-32 |
| 25 | 0.20 | 0.99 | 24-25 |
| 41 | 0.21 | 0.99 | 56-41 |
| 19 | 0.25 | 0.99 | 18-19 |
| 20 | 0.32 | 0.98 | 21-20 |
| 18 | 0.43 | 0.99 | 4-18 |
| 42 | 0.48 | 0.99 | 56-42 |
| 43 | 0.60 | 1.00 | 41-43 |
| 54 | 0.61 | 0.99 | 53-54 |
| 27 | 0.63 | 0.99 | 26-27 |
| 55 | 0.64 | 0.98 | 54-55 |
| 12 | 0.77 | 0.99 | 9-12 |
| 30 | 0.82 | 0.99 | 25-30 |
| 51 | 0.85 | 1.00 | 50-51 |
| 52 | 0.85 | 0.99 | 29-52 |
| 49 | 1.01 | 0.99 | 38-39 |
| 16 | 1.25 | 1.00 | 1-16 |
| 8 | 1.33 | 1.00 | 6-8 |
| 10 | 1.36 | 1.00 | 9-10 |
| 17 | 1.37 | 1.00 | 12-17 |
| 13 | 1.39 | 1.00 | 9-13 |
| 5 | 1.48 | 0.99 | 4-5 |
| 6 | 1.49 | 0.99 | 4-6 |
| 50 | 1.50 | 1.00 | 49-50 |
| 53 | 1.52 | 1.00 | 52-53 |
| 38 | 1.69 | 1.00 | 37-38 |
| 28 | 1.77 | 0.99 | 27-28 |
| 35 | 2.04 | 1.00 | 34-35 |
| 15 | 2.50 | 0.99 | 1-15 |
| 3 | 2.65 | 0.99 | 2-3 |
| 29 | 2.80 | 1.00 | 28-29 |
| 33 | 2.86 | 1.00 | 32-33 |
| 44 | 3.50 | 0.99 | 38-44 |
| 47 | 3.60 | 0.99 | 46-47 |
| 9 | 4.80 | 1.00 | 8-9 |
| 2 | 8.85 | 1.00 | 1-2 |
| 23 | 11.77 | 1.00 | 22-23 |

In this situation, rather than ensuring complete protection, the focus shifts to strategically placing additional PMUs at critical locations to mitigate hidden attacks from adversaries. The system's size and topology influence the required number of PMUs (Table 7).

**Table 7:** Solutions for various test systems: case 6

| Solution | Optimal Allocation of PMUs | | |
|---|---|---|---|
| | **IEEE-14** | **IEEE-30** | **IEEE-57** |
| | | $F_{Tcost}$ | |
| Optimal location of PMUs | 2,6,9 | 1,2,10,12,15,20 | 1,9,14,20,25,27,29, 32,41,51,54 |
| Average availability | 0.99 | 0.99 | 0.99 |
| $F_{Cost}$ | 99K | 180K | 331K |
| $F_{Tcost}$ | 126.72 | 189.48 | 359.87 |
| SORI | 16 | 43 | 61 |
| WBOI | 10 | 18 | 20 |
| BOI | 6 | 25 | 41 |
| No. of weak buses | 1 | 14 | 1 |
| | | $F_{FVSI}$ | |
| Optimal location of PMUs | 2,4,6,9,10,12,13,14 | 2,4,10,12,15,17,19,20,24,25,26,27,29 | 1,4,7,8,11,12,13,19, 20,24,25,26,27,29,31, 32,39,40,41,45,46, 50,51,52, 54,55, 56 |
| Average availability | 0.99 | 0.99 | 0.99 |
| $F_{Cost}$ | 214K | 347K | 708K |
| $F_{Tcost}$ | 194.77 | 487.70 | 1650.14 |
| SORI | 34 | 55 | 103 |
| WBOI | 29 | 41 | 76 |
| BOI | 5 | 14 | 27 |

To evaluate the performance of the proposed approach in the respective test systems, the number of branches connected to each bus is used as a metric. If a bus is linked to more than four branches, it is classified as a critical bus. As a result, additional PMUs need to be installed to address the loss of observability.

**Table 8:** MOPPMU (FTcost versus FFVSI) solutions for various test systems: case 7

| Solution | BCALO |
|---|---|
| **IEEE-14 Bus** | |
| Optimal location of PMUs | **2,6,9,10,13** |
| Average availability | 0.99 |
| $F_{Cost}$ | 139K |
| $F_{Tcost}$ | 414.70 |
| SORI | 22 |
| WBOI | 16 |
| BOI | 6 |
| Weak buses can monitor reliably for *F* | 6 |
| **IEEE-30 Bus** | |
| Optimal location of PMUs | **2,4,10,12,15,18,19,24,26, 27,29** |
| Average availability | 0.99 |
| $F_{Cost}$ | 295K |
| $F_{Tcost}$ | 498.68 |
| SORI | 74 |
| WBOI | 36 |
| BOI | 38 |
| Weak buses can monitor reliably for *F* | 11 |
| **IEEE-57 Bus** | |

| | |
|---|---|
| Optimal location of PMUs | **1,4,6,11,13,20,25,27,29,32, 38,39,40,44,46,52,54,56** |
| Average availability | 0.99 |
| $F_{Cost}$ | 493K |
| $F_{Tcost}$ | 987.47 |
| SORI | 79 |
| WBOI | 54 |
| BOI | 25 |
| Weak buses can monitor reliably for $F$ | 16 |

To mitigate this loss, the most critical locations should be made observable by at least two PMUs. Table 9 presents the deployment of additional PMUs at the relevant locations across different test systems.

**Table 9:** Results of defense management

| Test Systems | No. of additional PMUs | Optimal locations of PMUs | $F_{SORI}$ |
|---|---|---|---|
| IEEE-14 | 1 | 5 | 24 |
| IEEE-30 | 1 | 13 | 51 |
| NE 39 | 2 | 5,25 | 57 |
| IEEE-57 | 6 | 2, 3, 12, 38, 40, 56 | 93 |

## 7. Conclusion

The state estimation module is a crucial component of any engineering program, significantly impacting an individual's overall grade point average. This article presents an investigative overview of the OPP problem within a co-simulation framework using MATLAB and the ALO toolbox, beginning with the appropriate mathematical model. The ALO toolbox is utilised to evaluate the PMU placement issue under various operating conditions. Additionally, this article provides a structured approach to initiating a co-simulation framework by applying concepts from electrical engineering, particularly state estimation, to characterise PMUs. The inclusion of case studies across various test systems provides a practical evaluation, demonstrating the methodology's adaptability to different grid sizes and configurations. Furthermore, the focus on sustainability ensures long-term applicability and accounts for future grid expansions. While the results are promising, further validation across more complex, real-world systems will solidify its robustness and practical implementation in diverse power environments.

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